5. $48\frac{17}{29}\times 49\frac{12}{29}=2400\frac{697}{841}$; $1295\frac{57}{93}\times 1296\frac{36}{93}=1677615\frac{7343}{8649}$.

6. $7464\times 7536=56248704$; $88044\times 87956=7744000000-1936$.

7. $2777\frac{7}{9}\times 41666\frac{2}{3}\times 666\frac{2}{3}\times 54\times 24\times 52\times 7692307\frac{9}{13}\times 625\times 125\times 56\times 32\times 142857142857142\frac{4}{7}\times 2083\frac{1}{3}\times 48\times 833\frac{1}{3}\times 3125\times 68543764287590=\ldots$"

These and eleven other even longer computations are carried out mentally by Mr. Case.

The principle, $a^2=(a-b)(a+b)+b^2$, may be used in the squaring of any number, though it is not so readily used if the numbers consist of more than two digits. Thus,

$$87^2=(87-3)(87+3)+3^2=84\times 90+9,$$
$$92^2=(92-2)(92+2)+2^2=90\times 94+4.$$

This is the principle used in several of Mr. Case's calculutions. Thus, $(5\frac{1}{2})^2=(5\frac{1}{2}-\frac{1}{2})(5\frac{1}{2}+\frac{1}{2})+(\frac{1}{2})^2=30\frac{1}{4}$. ED. F.

330. Proposed by R. D. CARMICHAEL, Princeton, N. J.

An important function in the Theory of Numbers is one defined thus: $f(x)=1$ when $x>0$, $f(x)=0$ when $x=0$, $f(x)=-1$ when $x<0$. Two analytic expressions for $f(x)$ are the following:

$$f(x)=\lim_{n \doteq \infty} x^{1/(2n-1)},\ n=1,\ 2,\ \ldots;\quad f(x)=\lim_{n\doteq\infty}\frac{(x+1)^n-(x+1)^{-n}}{(x+1)^n+(x+1)^{-n}},\ x>-1.$$

It is required to find other non-trigonometric analytic expressions for this function. (There are several representations of $f(x)$ by means of trigonometric functions.)

Remark by the PROPOSER.

Professor F. H. Safford, of the University of Pennsylvania, has sent me the following expressions for the function defined in the problem:

$$\frac{2}{\pi}\int_0^\infty \frac{\sin xz}{z}dz,\quad \frac{2}{\pi}\int_0^\infty \frac{xdz}{x^2+z^2},\quad \operatorname*{Lim.}_{m=+\infty}\frac{e^{xm}-e^{-xm}}{e^{xm}+e^{-xm}}.$$

333. Proposed by R. D. CARMICHAEL, Princeton University.

Sum the infinite series

$$\frac{1}{(m+1)^2}+\frac{(2m-1)}{(2m+1)^2}+\frac{(3m-1)^2}{(3m+1)^4}+\frac{(4m-1)^3}{(4m+1)^5}+\frac{(5m-1)^4}{(5m+1)^6}+\ldots$$

[No solution of this problem has been received.]

334. Proposed by G. B. M. ZERR. A. M., Ph. D., Philadelphia, Pa.

Sum the series, $2^n-n\,2^{n-2}+\frac{n(n-3)}{2!}2^{n-4}-\frac{n(n-4)(n-5)}{3!}2^{n-6}+\frac{n(n-5)(n-6)(n-7)}{4!}2^{n-8}-\frac{n(n-6)(n-7)(n-8)(n-9)}{5!}2^{n-10}+\ldots$